

Hypothesis Testing

Below is a sample data set that we will be using for today's exercise. It lists the heights for 10 men and 12 women collected at Truman State University. The data will be entered in the same way that it was in the **Minitab introduction** handout. In addition to the "Height" and "Gender" columns, we will add another column "Hfemale". Click on the variable view tab and name the new column. Then return to the data view spreadsheet and enter the 12 heights for the females in this new column.

Male	Female
69	65
70	61
65	67
72	65
76	70
70	62
70	63
66	60
68	66
73	66
	65
	64

	C1	C2-T	C3	C4
	Height	Gender	Hfemale	
1	69	M	65	
2	70	M	61	
3	65	M	67	
4	72	M	65	
5	76	M	70	
6	70	M	62	
7	70	M	63	
8	66	M	60	
9	68	M	66	
10	73	M	66	

Analyzing data

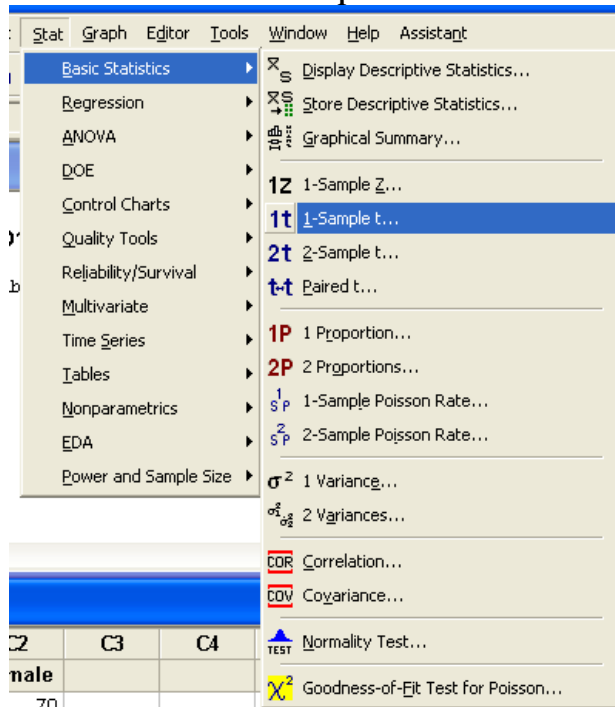
We commonly are interested in comparing the mean of the population to some known standard. We do this by creating a hypothesis, sampling from the population, calculating a test statistic from the sample, and then determining if the sample could reasonable come from the population if the null hypothesis was correct. Below are several examples illustrating various hypothesis tests.

2-tailed Hypothesis test

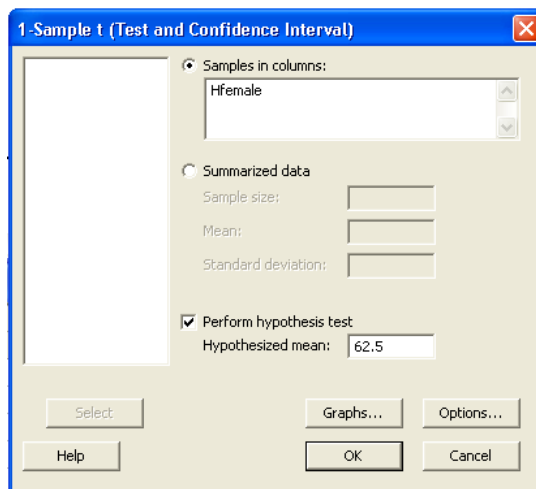
A researcher is interested in knowing if the average height for women at Truman State University is equal to 62.5 inches. The appropriate hypothesis are

$$H_o : \mu_F = 62.5 \text{ inches} \qquad H_A : \mu_F \neq 62.5 \text{ inches}$$

The appropriate test for this hypothesis is the one sample t-test. To run this test, click on **Stat - Basic Statistics - 1 Sample t**



This will bring up the 1-Sample t Test window. To run the test we need to move “Hfemale” into the test variable and change the Hypothesized mean to 62.5 [the value of the null hypothesis].



Minitab will not indicate whether the null hypothesis has been rejected. That is up to the researcher to determine from the presented results in the session window. Below is the output received from Minitab for the female heights. We will use the P-value method to determine if the null hypothesis should be rejected. The output contains the t value for the data and its degrees of freedom. Additionally, it displays the test value [under H_0] and a column labeled P.

The screenshot shows the Minitab Session window with the following content:

Variable	N	Mean	StDev	SE Mean	95% CI
Female	12	64.500	2.747	0.793	(62.755, 66.245)

One-Sample T: Hfemale

Test of $\mu = 62.5$ vs not = 62.5

Variable	N	Mean	StDev	SE Mean	95% CI	T	P
Hfemale	12	64.500	2.747	0.793	(62.755, 66.245)	2.52	0.028

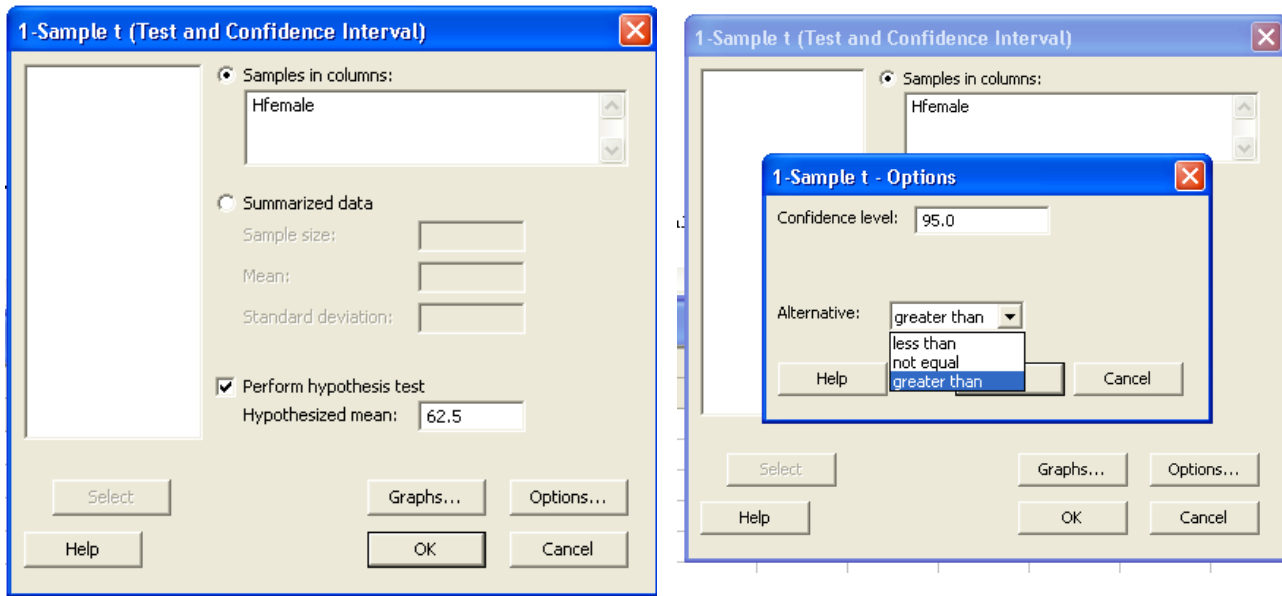
This number labeled “P” is what we refer to as the P-value. It is the probability of seeing a test score this extreme or more [in the direction of the alternative hypothesis] if the null hypothesis is true. The smaller the number, the more rare our test score is under H_0 and the more likely that the null hypothesis isn’t true. Using the .05 significance level as our cutoff, we find the P-value of .028 to be in our rejection region. We reject the null hypothesis and conclude that there is sufficient evidence that the heights of women at Truman are not equal to 62.5 inches.

One tailed hypothesis test (example 1)

A researcher believes that the female population at Truman State is taller than the average height of the general population of the United States [62.5 inches].

$$H_o : \mu_F = 62.5 \text{ inches} \qquad H_A : \mu_F > 62.5 \text{ inches}$$

We conduct the analysis on Minitab exactly as it was done before. The only difference will be in the **1-sample t (Test and Confidence Interval) Window**, select **Options**. This will bring up an window where the Confidence level can be changed. What we are really interested in this window is under the **Alternative** drop down box. **Not equal** should currently be selected but since this is a one sided t-test saying that the mean is greater than 62.5 inches, we want to select **greater than**. Click **OK** to exit out of this window and then **OK** on the 1-Sample t window. This will calculate our new p-value of .014.

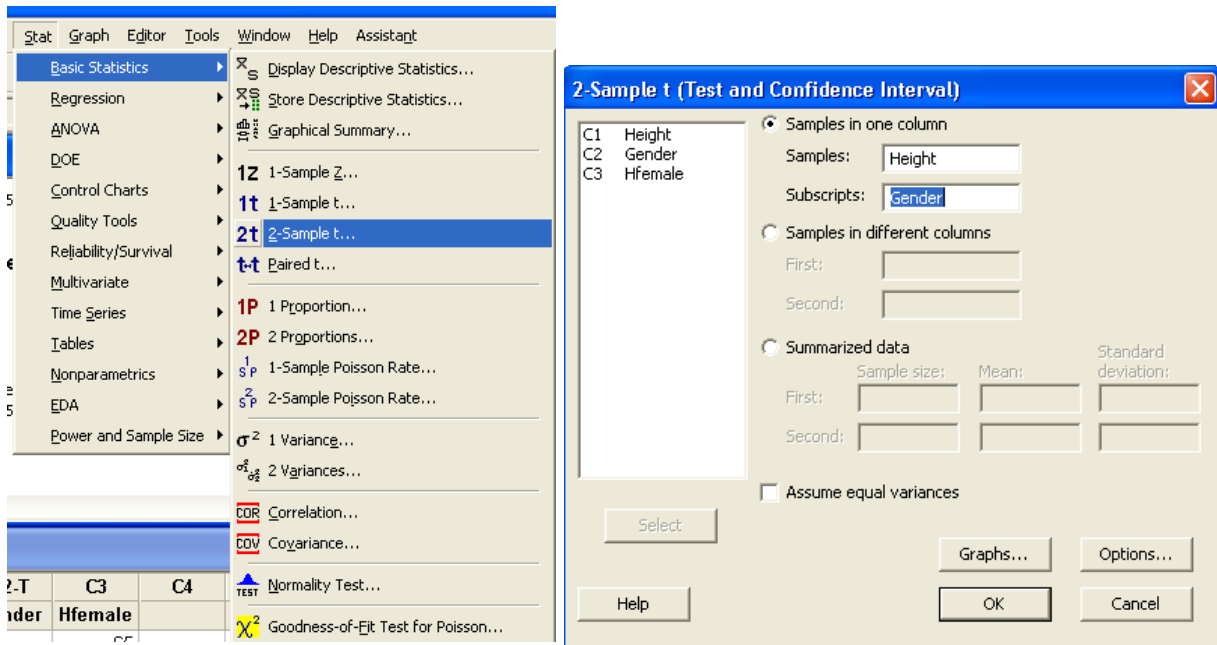


Comparing 2 independent samples

One question of interest in this data set might be to determine if there is a statistical difference in the heights of men and women. The appropriate hypotheses for this question is

$$H_o : \mu_M - \mu_F = 0 \qquad H_A : \mu_M - \mu_F \neq 0$$

The appropriate test for this hypothesis is the independent samples t-test. To run this test, click on **Stat – Basic Statistics – 2 Sample t**

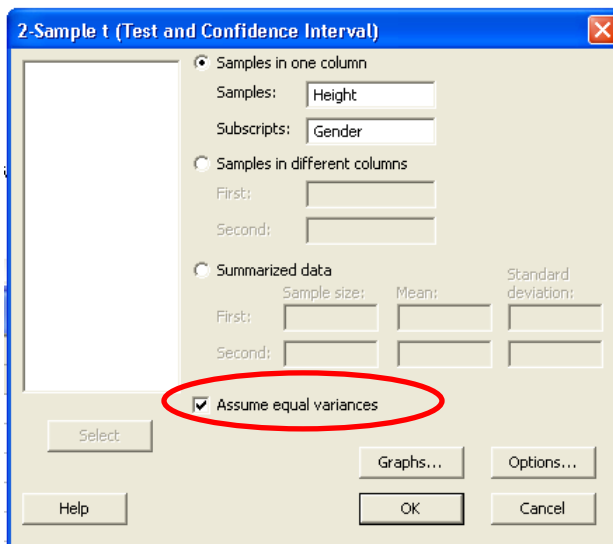


Hypothesis Testing

We then move the height and gender to their appropriate fields as seen below and then select **OK**.

The correct degrees of freedom Δ , which in this case is 17, is calculated by Minitab automatically. As before the table contains t-observed, the df, and the P-value for the test. We can see that we have a P-value of .001 so we reject the null hypothesis and conclude that there is sufficient evidence that the average height of men and women are not the same. This calculation is for unequal variance.

If we want to assume equal variances, we go through the exact same process except in the **2 Sample t** window, we need to check the box near the bottom that says **Assume equal variances**.



Now, our p-value is .000 (which is rounded to 3 decimal places so the true value could really be .0004) and our DF is 20.

If we check the calculation for df assuming unequal variances

$$\Delta = \frac{\left[\frac{3.25^2}{10} + \frac{2.75^2}{12} \right]^2}{\frac{\left(\frac{3.25^2}{10} \right)^2}{10-1} + \frac{\left(\frac{2.75^2}{12} \right)^2}{12-1}} = 17.77$$

We see that the hand calculations match the Minitab output. Note also that we receive the same conclusion regardless of which test we use, though in some cases they may lead to different conclusions.

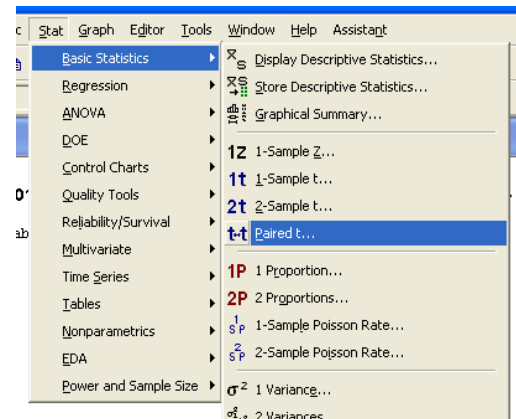
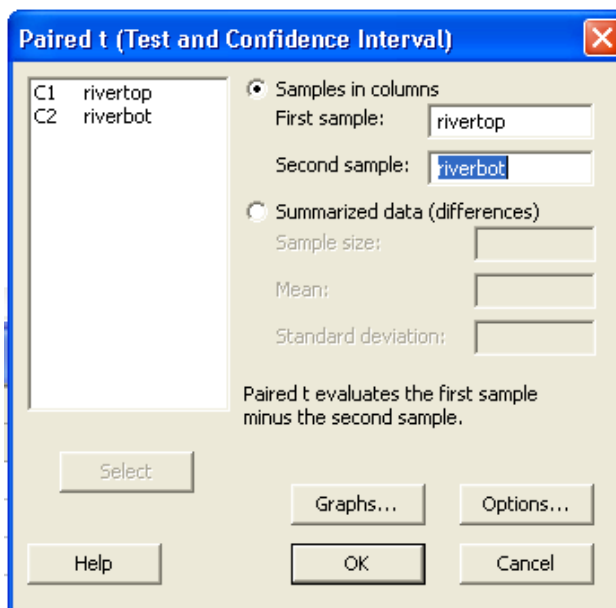
Comparing 2 dependent samples (Paired t-test)

If the test subjects are matched and paired by certain traits when assigned to the treatment groups, we no longer have independent samples. Now each measurement in a treatment group can be reasonably paired with a measurement in the other treatment group [and this was done before the experiment was run!]. The data will be placed into 2 columns with each row corresponding to a “paired” data set. For the river water quality example done in class we have 6 locations with a measurement at the top and bottom at each location.

Worksheet 1 ***			
↓	C1	C2	
	rivertop	riverbot	
1	0.415	0.430	
2	0.238	0.266	
3	0.390	0.567	
4	0.410	0.531	
5	0.605	0.707	
6	0.609	0.716	
7			

To test if there is a significant difference between top and bottom we run the paired Samples t test in Minitab.

This will bring up the data window. We place the variables in the first and second sample boxes (it doesn't matter which sample goes in which box).



Session

Paired T for rivertop - riverbot

	N	Mean	StDev	SE Mean
rivertop	6	0.4445	0.1418	0.0579
riverbot	6	0.5362	0.1713	0.0699
Difference	6	-0.0917	0.0607	0.0248

95% CI for mean difference: (-0.1554, -0.0280)

T-Test of mean difference = 0 (vs not = 0): T-Value = -3.70 P-Value = 0.014

We can see that the p value for the test is well below our cutoff of 0.05 and so we would reject the null hypothesis and conclude there is significant evidence that the concentration of pollutants differs from the top and bottom of the river.